

# **High-Temperature Mechanism-Based Design**

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# PHILOSOPHICAL TRANSACTIONS

# High-temperature mechanism-based design

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The historical development of the procedures used in high-temperature design using isotropic materials is described. It is illustrated how knowledge of the mechanisms causing time-dependent deformation and damage can help to describe the influence of the multi-axial states of stress occurring in practice and to provide rational procedures for extrapolating the results of short-term tests. Since the procedures are based on mechanisms, their structure lends itself readily to a concurrent engineering approach.

Procedures which support the application of composites to high-temperature designs are much less well developed but the progress to date supports the notion that mechanism-based design procedures are practical, economic and amenable to concurrent engineering.

### 1. Introduction

The procedures of design with monolithic metals are now well established having been in a state of constant development for over a century. The concept of ductility (Fairbairn 1856) enabled engineers, if only intuitively, to use plastic deformations to design efficient and reliable components. The experiments of Wöhler (1860) demonstrated that the application of many cycles of load could cause failure at levels which are fractions of those causing short-time failure. The theory of elasticity (Love 1892) provided the theoretical support for its practical application by means of strength of materials. The rules of design gradually developed from experience combined with the results of experiments on large complex components (Blair 1946). By the middle of the century, the theory of plasticity was established definitively (Hill 1950) and the results were being applied to civil engineering (Baker et al. 1956) and pressure vessel technology stimulated by the demands of nuclear power (Kennedi 1960). The combination of economic and technological factors encouraged the increased use of high-strength steels. Experience with the use of high-strength steels pointed to the importance on performance of defects introduced during the process of manufacture and construction. These circumstances stimulated renewed interest in the previous studies of Griffith (1920) and resulted in the theory of fracture (Irwin 1958) which is now widely practised as a design tool.

The availability of these concepts provides a box of design tools which are sharp enough to be used across a spectrum of applications from electronic components to offshore structures. Each application has its own set of nuances but the uniformity of language eases communication between the disciplines of materials science, design and manufacture. Design involves many decisions which

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are easier to make when supported by a small number of robust concepts. Their application must avoid complex procedures which encumber the creative process. As design progresses, more emphasis and effort may be expended on details and refinement, but these must be compatible with the overall thrust of the original design concept.

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The design concepts using the new breed of materials, such as those proposed for future aircraft engines remain to be established. There are several difficulties. Material development is characterized by rapid change so that only small amounts are available, for even the simplest tests. Specimens which duplicate the complex states of stress occurring under operating conditions are extremely expensive, even if sufficient material is available. Information from a build and test experience base will not be available. Drawing blindly from experience with monolithic metals is likely to result in failures, which are the consequence of unsuspected mechanisms. Under these circumstances, new design procedures must include the role of material mechanisms on component performance. This has been possible in high-temperature design with metals. The procedures described in R5 (1991) are simple, robust, science-based and provide a common language between materials scientist and designer. In this paper, an abbreviated account is given of attempts to establish design procedures for new high-temperature composite materials.

# 2. The mechanics and mechanisms of current high-temperature design

The mechanics of design for components which operate at high temperature are already in place and have been incorporated into working design codes (R5 1991). Because the life and deformation of high-temperature components are dictated by the time-dependent failure mechanisms, it has been essential that the results of materials science be integrated into the design process. Material properties establish the connection between materials science and the continuum mechanics as described in the text of Ashby & Jones (1980).

Mostly, but not always, components operate in the elastic range. The elastic moduli of the material which are the consequence of interatomic forces are the parameters needed to complete the elastic analysis of a component. Hence, the properties provide the means of relating the microscale to the scale of components according to the relation shown below.

mechanism -	→ property -	$\rightarrow$ continuum mechanics –	→ design concepts
interatomic force	E,G	theory of elasticity (Love 1892)	strength of materials
			$\begin{array}{c} \text{stress} \\ \text{concentration} \\ \text{factor} \end{array}$
			finite element

In addition to connecting scales, the properties provide information about the response to the multi-axial stress states occurring in practice. Hence, in the case of isotropic materials, both Young's modulus E and shear modulus G are required to

activate the theory of elasticity. Elasticity theory can, in principle, be used to solve problems. However, the theory of elasticity is an enormous intellectual edifice and it is the simplified methods known as strength of materials that designers use to estimate quickly, deformations and stress levels. Stress concentration values are also important in helping to determine design limits and these have been catalogued. For more complex, unusual problems, the finite-element method is a well-developed design tool.

An elastic response is normally expected when the component operates under normal working conditions. However, under unusual and infrequent operating conditions, it is common to exceed the conditions for which elasticity is valid. It is then necessary to include the influence of irreversible plastic deformations. The yield stress  $\sigma_y$ , is then the appropriate material property which relates dislocation mechanics to the continuum theory of plasticity as indicated below.

m	echanism $\longrightarrow$	$property \longrightarrow$	continuum mechanics -	→ design concepts
	islocation nechanics	$\sigma_{ m y}, J_2$	theory of plasticity (Hill 1950)	limit load bounds

The second stress tensor invariant  $J_2$  defines the influence of three-dimensional stress states. This is the simplest description and does not include important effects, such as work-hardening, but design procedures must be simple, and such concerns are left to a more detailed level of design. The application of the theory of plasticity has been a fertile ground for design because the application of the bounding theorems allows decisions to be made on the basis of quite simple calculations and can be part of the creative process.

Fracture has also been integrated into the design process and in addition to evaluating the influences of defects introduced during manufacture can also be used as a creative design tool. The relationship between mechanisms and design is indicated below.

mechanism	$\longrightarrow$ property $\longrightarrow$	continuum mechanics —	design concepts
voids	$K_{ m IC}$	fracture mechanics (Irwin 1958)	critical crack length
debonding	$K_{ m IIC}$		leak before break
	$K_{ m IIIC}$		test procedures

When two different mechanisms operate simultaneously, the conditions are set for the appearance of additional phenomena. For example, under cyclic loading conditions, when both elastic and plastic deformations occur, then the shakedown concept provides valuable insight into component behaviour. Comparison of the limit and fracture loads suggests the 'leak before break' concept defining the condition when contained fluid can leak before failure, providing additional safety to the design. The material parameter describing this mechanism parameter  $(K_{\rm IC}/\sigma_{\rm y})^2$ , which is a particular example of the material indices studied extensively by Ashby & Jones (1980).

The methods for high-temperature design fit into a similar framework by using

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Table 1. High-temperature design

identifier	$\longrightarrow$ mechanism	$\longrightarrow$ material $-$ property	→ continuum — mechanics	ightarrow design concepts
creep deformation maps	diffusion dislocation	$\sigma_{ m D}$ $\phi$	theory of creep	bounding methods
				modification of limit load shakedown
creep rupture maps	void growth	$\sigma_{ m R}$	continuum damage mechanics	bounding methods
creep ductility index	$\begin{array}{c} \text{microstructural} \\ \text{growth} \end{array}$	$\Delta$		

the reference stress as a material property which removes time from the design process. The deformation reference stress,  $\sigma_{\rm D}$ , is defined usually as that stress which, when applied to a uniaxial specimen for time equal to the design life causes a strain of 1\%. The rupture stress is that uniaxial stress which causes rupture at the design life. The need to extrapolate data and identify mechanisms necessitates the addition of a column headed Identifier so that the chart for high temperature is shown in table 1.

The effect of multi-axial states of stress on creep deformation is defined by the function  $\phi$ . The function  $\phi$  is usually taken to be the second stress invariant  $J_2$ , but under conditions dominated by diffusion  $\phi$  is the maximum stress. The studies of deformation creep mechanics have identified deformation bounds, so that deformations can be estimated for constant and cyclic loads and temperature. As it is for plasticity theory, the bounding principles are the basis of design procedures. When the function  $\phi = J_2$ , many of the calculations for plasticity can be used directly with the reference deformation stress  $\sigma_{\rm D}$  replacing the yield stress  $\sigma_{\rm D}$ . This means that the extension of the design methods of plasticity can be extended to creep and this is the basis of the R5 procedure.

The mechanics which describes component behaviour when the material deteriorates with time is known as continuum damage mechanics. This subject is of more recent origin and it is only recently that the first text on the subject has been published by Lemaitre (1992). The function  $\Delta$ , which describes the effect of three-dimensional stress states on damage growth, is dependent on the underlying damage mechanism. In diffusional creep rupture,  $\Delta$  tends to be the maximum stress,  $\sigma_{\rm max}$ , whereas for materials with changing internal dislocation state,  $\Delta$  is the second stress invariant  $J_2$ . The functional form of  $\phi$  and  $\Delta$  can be determined from experiment, but the tests are demanding, expensive and time consuming. Clearly, the ability to define  $\Delta$  from a knowledge of the underlying mechanism offers substantial benefits. This need has led to the interest in mechanism identifiers. The most commonly used are the high-temperature mechanism maps (Frost & Ashby 1982), which identify the operating limits of different damage mechanisms. Another identifier is the creep ductility index which is defined as the creep strain at rupture divided by the product of the steady-state creep

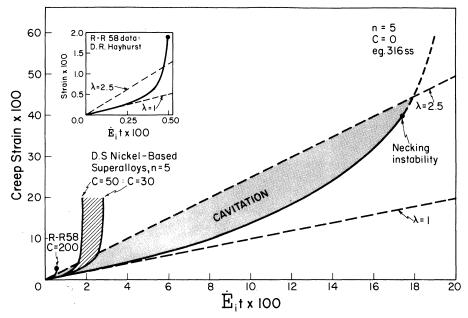


Figure 1. Diagnostic diagram.

rate and the time to rupture. The deformation is illustrated in figure 1. Dyson & Leckie (1988) have shown that, when  $\lambda \approx 1$ , the mechanism is diffusion controlled and  $\Delta = \sigma_{\text{max}}$ . When  $\lambda$  is large ( $\lambda = 10$ ), then  $\Delta = J_2$ .

It is a constant problem in high-temperature design that the experiments are accelerated and failure times measured in the laboratory are orders of magnitude less than the design life. It is then necessary to use some form of extrapolation to estimate the reference stress corresponding to the life of the component. A knowledge of the mechanism and the conditions under which they operate can direct an efficient test program and add confidence to the extrapolation techniques. Knowledge of the mechanism also helps to provide a mathematical description needed for design calculations. Finally, the mechanism growth laws can be expressed in terms of dimensionless groups. It has been established by Frost & Ashby (1982) that data obtained for one material may be used to describe the behaviour of another, provided they fall within defined material groupings. This implies that the results of long-term tests obtained for one material may be modified to provide information of another material within the same classification for which test data are not available.

Codes of practice define those design procedures which result in a reliable and functioning product without the need for expensive build and test verification. This latter requirement is especially important in complex systems. The most recent attempt to define design procedures, for high temperature operation comes from the Assessment Procedures R5 from Nuclear Electric plc (1991). The procedures are based on the framework described above, relating mechanisms and mechanics by means of the reference stress. Extensive use is made of bounding theorems, so that only elastic analysis is required. In this way, complex and time-consuming analyses, which impede design decisions, can be avoided. The overall

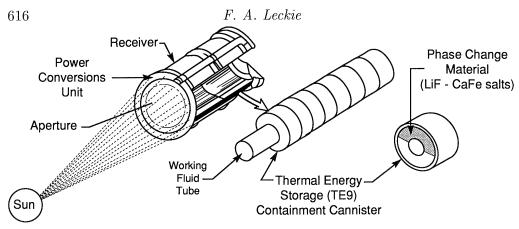


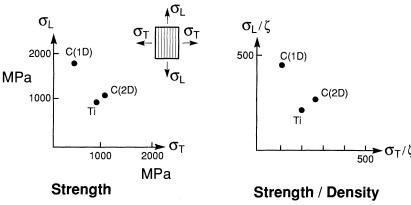
Figure 2. Space Station Freedom solar dynamic power module.

clarity of the approach also provides the flexibility necessary for integrating the effort needed to solve complex technological problems. The design methods have been formulated with large mechanical components in mind which operate up to  $600\,^{\circ}\mathrm{C}$ . The procedures used to design aircraft engines are the closely guarded results of industrial competitors, but it is clear in discussions with designers that the same ingredients govern the design process.

An illustration of the flexibility of the R5 procedure is the design of the power plant for a space station to circle the Earth for 30 years. The proposal uses LiF-CaFe salt, as the heat source of a Brayton thermodynamic cycle. When the spacecraft is facing the Sun, the rays are directed onto pellets, containing the salt eutectic which melts at 750 °C (figure 2). When the spacecraft passes behind the Earth away from the Sun, the melted salts solidify and supply the latent heat required to operate the power system during the darkened portion of the flight while maintaining a constant temperature heat source. The design life is 30 years, the cycle time around the Earth is 90 min, so that there are 175 000 operating cycles. The pressure of the working fluid is 308 kPa and operates at 750 °C. Upset conditions include the plant shutdown, followed by restart when the a temperature increase is 1000 °C. The purposed material is the cadmium alloy Haynes 88. Following the systematic interpretation of the R5 procedure reveals that the most critical mechanism is the creep rupture of the containment vessel operating at 750 °C. An instinctive reaction might have suggested that it is the severe upset conditions which are critical. It is an advantage of the R5 procedure that there is no prejudgement of the failure mechanism and each must be systematically investigated. The biggest unknown is long-term creep strength and the need to extrapolate rupture tests lasting 30000 h to the 263000 h life of the component. It is then that the mechanism approach allows rational and effective interpretation of available information and the selection of the reference stress.

### 3. Some observations on the use of composite material

In addition to the ability to operate at high temperatures, weight saving is an important design consideration for aero-engines so that the strength/density ratio features in material selections. However, since multi-axial stress states are the



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Figure 3. Bi-axial strength.

normal conditions in engineering components, the axial strength/density values may be an insufficient indicator of material behaviour.

The longitudinal transverse strength and strength/density properties of the MMC SiC-Ti, with unidirectional continuous fibres, are shown in figure 3a, b. The strength in the fibre direction is  $\sigma_{\rm L}$  and  $\sigma_{\rm T}$  in the transverse direction. Also plotted in the diagram are the strength and the strength/density values of the metal matrix. It can be readily seen that the addition of fibres to the titanium matrix improves the strength of the matrix in the fibre direction, but reduces it in the transverse. However, in circumstances when the stress state is uniformly biaxial and there is equal reinforcement in two directions, the resulting strength and strength/density values indicated in figure 3 are scarcely better than those of the matrix. The message is clear. To gain advantage of the strength/weight properties of the MMC, the loading anisotropy should match that of the material. An appropriate application of MMCs is the boron-aluminum uniaxial members proposed for a space station (figure 4). The selection of a monolithic material for the joints is appropriate on two counts. The stress states in the joint are multi-axial so there is no strength/density advantage to be gained by using a composite. Furthermore, the technology of monolithic joints is much more advanced. The interface properties between composite and monolithic are a new unknown introduced into the problem. The struts proposed to position nozzles in aero-engines is also illustrated in figure 4, with the SiC-Ti composite being bonded to a monolithic titanium joint. When designing the joint, the load is assumed to be transferred entirely in shear between the join and MMC. It is assumed no stress is carried across the normal interface. This assumption results in heavier joints and creates interfaces which may fail by fatigue.

Another application to be discussed later is the composite ring of an aeroengine, compressor, in which the dominant stress is in the hoop direction (figure 4). The centrifugal loads introduce transverse stresses into the ring which must be taken into consideration when attempting to determine the flow of the load into the fibres in the circumferential direction.

A basic concern is the decrease of composite performance in the presence of the stress concentrations which occur at holes and intersections. Experiments performed by Connell *et al.* (1994) on SiC–Ti MMC plates indicate that the introduction at holes and slits greatly reduce strength. The tensile strength of the

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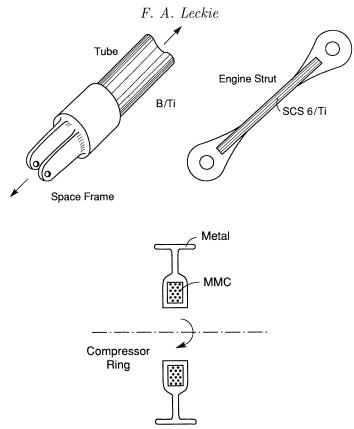
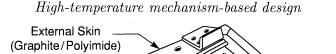


Figure 4. Typical metal-matrix composite component.

composite is 1800 MPa and the introduction of a 1 mm diameter hole reduces the strength to 1100 MPa. This result is consistent with a toughness of approximately 100 MPa, which agrees quite closely with the results of the detailed analysis by Connell et al. (1994). The design implication is evident. In the longitudinal direction the MMC is brittle, so that stress concentration must be avoided or special attention given to reinforcement. It is of little surprise then, that current applications appear to be limited to simple shapes which avoid the stress concentrations associated with holes or intersections. Similar experiments on panels of CMCs with notches (Cady 1994) indicate that these materials are tough and notch insensitive so that failure occurs when the average stress at the point of minimum cross section equals the ultimate strength of the CMC. During such tests, it is observed that there is multiple matrix cracking, which is the likely source of the negligible effect of stress concentration. CMCs must have multi-directional reinforcement and since they are notch insensitive, the presence of stress concentrations is of little importance. The CMC component shown in figure 5 is the exhaust chamber of an aero-engine. The mechanical load from internal pressure is small but the component has a complex shape with many points of stress concentrations.

The MMC rotor ring and the CMC exhaust chamber represent examples of the application of composites to specialized aero-engine components. Other presentations in these proceedings deal with ceramic matrix composites, so that further discussion is restricted to metal matrix composites.



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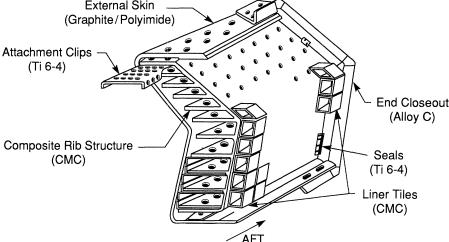


Figure 5. Typical ceramic-matrix composite component.

### 4. Behaviour of metal matrix composites

The behaviour of MMCs is strongly dependent on the strength of the interface between the fibre and matrix. The MMC SiC-Ti usually has a weak interface, while Al<sub>2</sub>O<sub>3</sub>-Al has a strong interface. A weak interface ensures that the strength of the fibres can be fully utilized, but this is achieved at the expense of transverse properties. By contrast, the MMCs with strong interface have relatively poor longitudinal strength but good transverse properties.

Because the effective use of MMCs implies unidirectional reinforcement, it is this form of reinforcement which is considered. In practice however, the operating stress states are multi-axial and it is necessary to determine the modes of deformation and failure for all stress states. Since the availability of materials under development is usually limited to a small number of small plates, it is not possible to perform the number or kind of tests used to characterize isotropic metals. A compensation is that the microstructure is usually well defined and the results of computation using homogenization methods can help to investigate stress states which are difficult experimentally (Gunawardeena et al. 1993). Although the properties of MMCs have been investigated in the fibre direction, perpendicular to the fibres and in shear, only the effect of transverse stress is covered for the present. It is the response to this loading which influences the behaviour of the rotor ring discussed later.

The maximum stress observed in the transverse stress-strain relationship (figure 6) for the SiC-Ti composite reaches only 40% of the strength of the matrix. The weak fibre-matrix interface is the source of this behaviour. There is thermal mismatch between fibre and matrix, so that the HIPing process used in production introduces compressive thermal stresses across the interface. On application of the transverse stress, the compressive interface stress is reduced until the residual stresses are overcome when the interface debonds. The presence of the debond decreases the transverse elastic modulus and strength, since the composite now behaves as titanium penetrated by holes the size of the fibres. These features are seen in the transverse stress-strain curve shown in figure 6, with debond occurring when the applied stress is 200 MPa. The reduction in the elastic modulus is 620

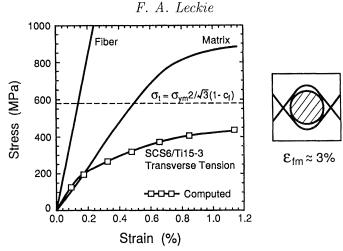


Figure 6. Transverse stress-strain relationship for SCS6-Ti composite.

substantial and the strength is only 40% of the strength of the matrix. Since there is no build-up of triaxial stresses, the failure strain of 1% is not dramatically less than the 3% observed for the matrix material.

### 5. Rotor design

Because of the high mechanical loads, the rotor of the compressor presents a demanding design problem. By replacing the conventional rotor by an MMC ring, it has been suggested that the weight of the compressor may be decreased by as much as 50%. A first attempt of a rotor design for study purposes is shown in figure 7. The rotor consists of a SiC-Ti composite ring in a Ti cladding. There are 25 blades attached to the outer diameter. The rotor speed is 1900 r.p.m. and the total centrifugal load from the blades is 1.3 MN.

The rings are manufactured using the HIPing procedure at 900 °C. Because of the thermal mismatch between fibre and matrix and the MMC ring and the cladding, thermal residual stresses are introduced into the microstructure of the composite and the ring.

The distribution of blade loading is not uniform along the perimeter and introduces troublesome multi-axial stress states into the MMC ring. This difficulty can be avoided by noting the load can be divided into components, one of which is uniform and equal to the average load. The other loading is periodic, being alternatively positive and negative, repeating itself over a distance equal to the circumferential pitch l. When the loading has the periodic form indicated in figure 8, it is known from elasticity theory that at a depth equal to the circumferential pitch l the stress has been reduced to less than 5% of the magnitude of the periodic loading component. Hence, provided the distance between the blades and the MMC ring is greater than the circumferential pitch of the blades the load applied to the ring is uniform and the irregularities of blade loading removed. It can be seen in the present design that the blade spacing is substantially less than the distance between blades and ring so that an immediate weight gain should be possible.

To understand how the centrifugal blade load is transferred into the ring, constitutive equations have been developed which describe the behaviour of the SiC-Ti

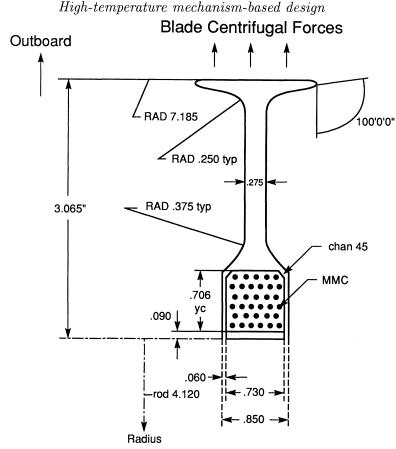


Figure 7. Compressor disc with MMC ring.

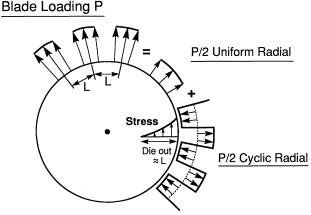


Figure 8. Components of blade loading.

MMC and can be used in finite-element calculations. The constitutive equations take into consideration the changes in the elastic and plastic deformations following debonding at the fibre—matrix interface.

The tensile stresses in the fibre direction are shown in figure 9. Because of the debonding (the debond angle is also shown in figure 9), the radial loading does

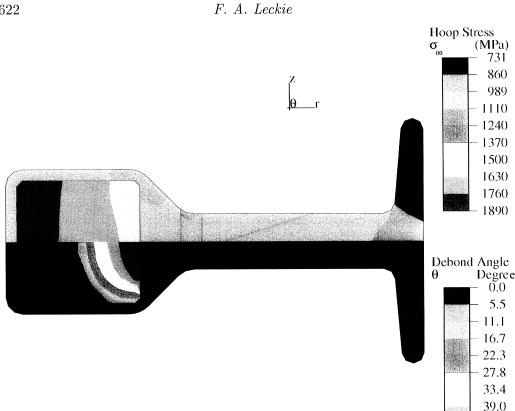


Figure 9. Hoop stresses and debond angle.

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not pass smoothly into the ring with some of the load being transferred down the outside cladding. Consequently, the tensile stress in the MMC ring is not uniform, being a maximum at the bore. This lack of uniformity in stress means that the ring is not used efficiently. Furthermore, when fibres begin to break, failure is catastrophic, which is clearly undesirable. By adjusting the shape of the crosssection, it is possible to achieve more uniform stress in the ring and at the same time, arrange that substantial plastic deformation occurs in the metal ligament before fibre failure. With this arrangement, plastic deformation occurs before the onset of general failure, thereby introducing a safety mechanism.

The constitutive equations are complex and need the results of non-standard tests together with micro-mechanics models. Their application is unlikely to appeal to designers and for that reason, something simpler is required. It has been observed in this study that the most important influence is the effect of debond of the fibre-matrix interface on the elastic properties. Hence, elastic calculations which at the outset assume the elastic properties of the MMC with debonded interface give a good indication of how the flow of stress supports the transverse loads. These calculations are very much simpler than those which follow the debonding process. Apart from any computational advantage, it is prudent to assume that the compression stresses following processing are eliminated by small amounts of creep so that the continuity of the fibre–matrix interface is broken and transverse properties are reduced.

### 6. Summary and conclusions

The design methodology for metals operating at high temperature is well developed. The procedures are based on the results of continuum mechanics and the role of materials is defined by a small number of material properties and functions which describe the behaviour under three-dimensional stress states. When the temperatures are such that time-dependent phenomena dominate, the selection of material properties and the dependence on multi-axial stress are dictated by the physical mechanisms. The organizational table 1 provides a comprehensive understanding of the role of each mechanism and the framework needed for concurrent engineering. High-temperature mechanics is nonlinear and the calculations required to predict component behaviour are expensive, lengthy and impede design decisions. Codes based on bounding methods have been formulated on the basis of elastic calculations which are completed with much greater ease.

A comparable framework for high-temperature design with composite materials is emerging but the material properties are dependent on the mix and geometry of the constituents on the composites. The increased choice adds complexity and emphasizes the need for a mechanisms-based methodology which increases understanding and avoids crushing knowledge overload. Some examples have been covered which show that it is possible to connect mechanism indicators with component performance. Generally speaking, understanding of the mechanisms is improving and the procedures are available formulate constitutive equations which provide faithful representation of the controlling mechanisms. However, the calculations are complex and indicate the need for simple procedures which can be used in design.

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